Prijemi ispit iz matematike septembar 2020

(1) Uprostiti izraz

$$\frac{1}{\frac{a}{a-2b}-\frac{2b}{a+2b}}\cdot\frac{a^2+4b^2}{a^2-4b^2},\qquad a\neq\pm 2b.$$

(2) Rešiti jednačinu

$$3^{2x}-12\cdot 3^x+27=0.$$

(3) Rešiti jednačinu

$$\log_3 x-\log_9 x+\log_{81} x=\frac{3}{4}.$$

Uputstvo. Npr. $\log_b a=\frac{\log_3 b}{\log_3 b}$ ili $\log_{x^a} y=\frac{1}{a}\log_a y$.

(4) Dokazati identitet

$$\frac{1+\cos\alpha+\cos 2\alpha}{\sin\alpha+\sin 2\alpha}=\cot\alpha.$$

SEPTEMBAR

1.

$$\frac{a^2-4b^2}{a^2+2ab-2ab+4b^2}\cdot\frac{a^2+4b^2}{a^2-4b^2}=1.$$

2.

$$t=3^x,\quad t^2-12t+27=0,\quad t_1=3,\quad t_2=9,\quad x_1=1,\quad x_2=2.$$

3.

$$\frac{\log_3 x}{\log_3 3}-\frac{\log_3 x}{\log_3 9}+\frac{\log_3 x}{\log_3 81}=\frac{3}{4}.$$

$$\log_3 x\cdot\left(1-\frac{1}{2}+\frac{1}{4}\right)=\frac{3}{4},\quad \log_3 x=1,\quad x=3.$$

4.

$$\begin{aligned}\frac{1+\cos\alpha+\cos 2\alpha}{\sin\alpha+\sin 2\alpha} &= \frac{\cos^2\alpha+\sin^2\alpha+\cos\alpha+\cos^2\alpha-\sin^2\alpha}{\sin\alpha+2\sin\alpha\cos\alpha}\\ &= \frac{\cos\alpha(1+2\cos\alpha)}{\sin\alpha(1+2\cos\alpha)}\\ &= \cot\alpha.\end{aligned}$$